A black hole solution to the cosmological monopole problem

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We propose a solution to the cosmological monopole problem: Primordial black holes, produced in the early universe, can accrete magnetic monopoles before the relics dominate the energy density of the universe. These small black holes quickly evaporate and thereby convert most of the monopole energy density into radiation. We estimate the range of parameters for which this solution is possible: under very conservative assumptions we find that the black hole mass must be less than 10^9gm.

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Magnetic monopoles are a generic prediction of Grand Unified Theories (GUTs). Monopoles always appear as stable topological entities in any GUT that breaks down to electromagnetism [1,2]. The monopole mass is $M_m \sim \eta/e$, where η is the energy scale of the phase transition and $1/e = \sqrt{137}$. Hence, for $\eta \simeq 10^{15}$ GeV, monopoles of mass $M_m \simeq 10^{16}$ GeV should exist. We use this mass scale as our fiducial value throughout, for the sake of definiteness, but note that an analogous calculation can be done for any other scale*. The magnetic charge of the monopoles, g, is determined by the quantization condition $eg = n/2$ (n is an integer). The abundance of these monopoles is an open question. The Kibble mechanism [3] predicts roughly one monopole per horizon volume at the time of the GUT phase transition (due to causality constraints). However, this estimate provides a severe overabundance of the number of monopoles: they overclose the Universe by many orders of magnitude. Since they are stable for topological reasons, they do not decay; it has also been shown that the rate of pair annihilation cannot reduce the monopole number sufficiently [4]. Their abundance is expected to be too high to be consistent with astrophysical observations of magnetic fields in our Galaxy [5] and consequences of catalyzed nucleon decay in stars [6,7]. This overabundance of GUT monopoles is known as the monopole problem.

An inflationary epoch [8] may reduce the density of monopoles in the Universe (for some alternative approaches see e.g. [9]). The rapid expansion phase dilutes the monopole abundance. The re-heating temperature must be sufficiently low in order not to produce the monopoles again. On the other hand, since any initial baryon-antibaryon asymmetry will be also washed out, the re-heating temperature must be sufficiently high in order to provide a mechanism for the observed baryon-antibaryon asymmetry. Here we explore new solutions to the monopole problem which, for example, would obviate the need for a low re-heating temperature.

As the universe expands, the cosmological horizon grows to encompass many monopoles. The predicted initial ratio [10] between the monopole and entropy density is given by

$$\left(\frac{n_m}{s}\right)_i \sim p g_*^{1/2} (\eta/M_{Pl})^3 \sim 10^{-12}\,, \tag{1}$$

where $p \sim 0.1$ is a geometrical factor and $g_* \sim 100$ is the number of effectively massless degrees of freedom at a given temperature. This ratio would remain constant as the universe adiabatically expands. However non-adiabatic processes, such as monopole/antimonopole annihilation or accretion by black holes, reduce this ratio.

Here we first review the relic monopole abundance subsequent to annihilation effects only, and later include a new effect: the fact that primordial black holes can eat monopoles. The relic abundance is far too large with annihilation only, but this problem can be resolved with black holes.

The motion of a monopole in a plasma is basically a random walk where the typical step is the mean free path. During each step a monopole moves with thermal velocity $v_T = (2T/M_m)^{1/2}$. The mean free path l depends on the scattering of a monopole on charged particles in plasma and is given by [10]:

$$l \approx \frac{M_m^{1/2}}{bT^{3/2}}\,. \tag{2}$$

The factor b depends on the number of degrees of freedom of light charged particles and its value is $b \sim 100$. Monopole-antimonopole capture occurs when $g^2/r \sim T$, which gives the capture radius $r_c \sim g^2/T$. Annihilation effectively takes place as long as the mean free path of a monopole is smaller than the capture radius for annihilation [11]. This implies that the annihilation practically stops at the final temperature

$$T_f^{\rm ann} \sim \frac{M_m}{g^4 b} \sim 10^{11}{\rm GeV}\,, \tag{3}$$

and the monopole to entropy density ratio freezes at the final value [4,10]

$$\left(\frac{n_m}{s}\right)_f \sim 1/(g^6 b g_*^{1/2}) M_m/M_{Pl} \sim 10^{-10}\,. \tag{4}$$

*For example, for $\eta \sim 10^{17}$GeV (which is favored by supersymmetric extensions of the standard model and by experimental constraints on the proton lifetime), the monopole mass would be 10^{18}GeV.

The annihilation mechanism is effective only if the initial monopole to entropy density ratio is greater than the one given by (4). Otherwise, the initial ratio remains constant as the universe expands. For our fiducial choice of parameters with $M_m \sim 10^{16}$ GeV, annihilation is irrelevant and the initial monopole to entropy density ratio of Eq.(1) remains constant; for other parameters and monopole masses, however, Eq.(4) should be used. The entropy density at a given temperature T is $s \sim g_* T^3$. The total mass of monopoles within the Hubble volume is thus

$$M_{\rm tot} \sim \left(\frac{n_m}{s}\right)_f sV_h M_m \approx 10^{-2}\left(\frac{n_m}{s}\right)_i \left(\frac{M_{Pl}}{T}\right)^3 M_m \,, \tag{5}$$

where $V_h \sim \frac{4\pi}{3}\tau_h^3$ is a Hubble volume at a given Hubble time $\tau_h = 0.3 g_*^{-1/2} M_{Pl}/T^2$. This mass will equal the total mass within the Hubble volume ($M_h \sim 0.3 g_*^{-1/2} M_{Pl}^3/T^2$) for $T \sim 10^3$GeV. Thus, monopoles dominate the energy density of the Universe at $T < 10^3$GeV. Thus the predicted present day monopole flux grossly violate many late-time cosmological constraints, if one considers annihilation as the only mechanism for reducing the monopole number.

Now we will consider the effects of black holes in reducing the monopole number. The early universe can produce large numbers of primordial black holes, via a number of processes [12–15]. The earliest mechanism for black hole production can be fluctuations in the space-time metric at the Planck epoch. Large number of primordial black holes can also be produced by nonlinear density fluctuations due to oscillations of some (scalar) field. If within some region of space density fluctuations are large, so that the gravitational force overcomes the pressure, we can expect the whole region to collapse and form a black hole. Black holes can also be produced at first and second order phase transitions in the early universe [14]. Collapse of cosmic string loops and closed domain walls also yields black holes.

We will show that black holes could solve the monopole problem by capturing them before they become dangerous for the standard cosmology. Since the black holes evaporate, much of the dangerous excess monopole energy density would then be converted into harmless radiation. If the universe contains enough black holes to remove monopoles quickly enough, but not so many of them to cause deviations from standard cosmology, then the monopole problem would be solved.

Primordial black holes can have a variety of masses. A typical mass would be that of the mass inside the horizon at the time of formation, so that the mass would range roughly from M_{Pl} (black holes formed at the Planck epoch) to M_{sun} (black holes formed at the QCD phase transition). However, the mass could be much smaller [13]. For example, black holes formed by collapsing closed domain walls and string loops are typically much lighter than a horizon mass. After formation black holes can evolve further by evaporating, merging with each other and accreting surrounding matter. Since the number of black holes of a given mass within a horizon volume as a function of time is strongly model dependent, we do not limit ourselves to any particular model. We instead solve the dynamical equation for the time evolution of the number density of monopoles with free parameters that determine the abundance of black holes, and then estimate the range of these parameters that is required for the solution of the monopole problem. We will show that black holes of mass $M_{bh} < 10^9$gm (a mass range that does not usually put any serious constraints on standard cosmology) are dynamically capable of removing the monopoles.

Monopoles move with non-relativistic velocities $v_M << c$ in the early universe, since they are magnetically charged and their velocity is damped due to interaction with surrounding plasma. The cross section for gravitational capture of a non-relativistic massive particle (monopole) by a black hole of gravitational radius R_{bh} is [16]:

$$\sigma_g = 4\pi (c/v_M)^2 R_{bh}^2. \tag{6}$$

The monopole flux is given by $n_m v_m$, where n_m is the number density. If there is only one black hole in the Hubble volume, the number of monopoles that get captured by the black hole, during the Hubble time τ_h, is $n_m v_m \sigma_g \tau_h$. The total number of captured monopoles is obtained by multiplying this quantity by the number of black holes within the Hubble volume, $n_{bh}V_h$, where V_h is the Hubble volume. This number should be compared with the total number of monopoles contained within the Hubble volume, $n_m V_h$, thus giving the condition

$$n_{bh}\sigma_g v_m \tau_h > 1 \,, \tag{7}$$

This inequality represents the condition under which the characteristic time for monopole capture is shorter than the Hubble time. We will now ascertain the resulting number density of monopoles by examining the dynamics in more detail. As we will see, the term on the left hand side of (7) will play the role of the suppression factor for the evolution of the monopole number density. How big this factor should be in order to solve the monopole problem, we will see from the equation that describes the dynamics of the process.

The dynamical equation (i.e. the detailed balance equation) for the time evolution of number density of monopoles in comoving volume is

$$\frac{dn_m}{dt} = -An_m^2 - n_{bh}\sigma_g v_m n_m - 3\frac{\dot{a}}{a}n_m \,. \tag{8}$$

The first term on the right hand side is present as long as the monopole-antimonopole annihilation is taking place. The parameter A was estimated in [11] as

$A = g^2/(bT^2) = g^2 g_*^{1/2} t/(0.3bM_{Pl}) \approx 10t/M_{Pl}$. The second term takes into account the monopole capture by the black holes while the third term is due to expansion of the universe (a is the scale factor of the universe).

We now examine the term $n_{bh}\sigma_g v_m$. The energy density of the black holes at any given time must be less than the energy density of the universe, so that $\rho_{bh} = M_{bh}n_{bh} \sim f g_* T^4$, where $f \leq 1$, while M_{bh} is the mass of a typical black hole at that scale. The number density of black holes is thus $n_{bh} \sim f g_* T^4/M_{bh}$. For a given black hole mass M_{bh}, the radius $R_{bh} = 2M_{bh}/M_{Pl}^2$. The relative black hole-monopole velocity is just a monopole random walk velocity $v_m = v_T/\sqrt{N}$, where v_T is a thermal velocity while N is the number of random walk steps. At some distance r, we have $\sqrt{N} = r/l$, where l is the length of the step (a mean free path). The characteristic distance is the gravitational capture radius calculated from the condition $M_{bh}M_m/(M_{Pl}^2 r_c) \sim T$. This gives $v_M = \sqrt{2}M_{Pl}^2/(bM_{bh}M_m)$. The mass of a typical black hole is just a fraction of the mass within the horizon at any given time. In other words,

$$M_{bh} \sim \beta\, 0.3\, g_*^{-1/2} \frac{M_{Pl}^3}{T^2}, \tag{9}$$

where $\beta \leq 1$. In the absence of complete knowledge of the distribution of black holes as a function of time, an assumption must be made about the time dependence of f and β. In our analysis of the dynamics, we will assume that the parameters f and β do not change significantly with time as they describe some average features of the system[†]. Substituting these relations into $n_{bh}\sigma_g v_m$ we obtain

$$n_{bh}\sigma_g v_m = f\beta^2 b M_m \approx 0.1 f\beta^2 M_{Pl}\,. \tag{10}$$

The dynamical equation (8) now reads

$$\frac{dn_m}{dt} = -\frac{10t}{M_{Pl}} n_m^2 - 0.1 f\beta^2 M_{Pl} n_m - \frac{3}{2t} n_m\,. \tag{11}$$

The solution to this equation is

$$n(t) = \frac{e^{-0.1 f\beta^2 t/t_{Pl}}}{C_1 t^{3/2} + (0.1\pi/M_{Pl}^3 f\beta^2)^{1/2} t^{3/2} \mathrm{erf}\left(\sqrt{0.1 f\beta^2 t}\right)}\,, \tag{12}$$

where $\mathrm{erf}(x) = 2/\sqrt{\pi}\int_0^x e^{-t^2}dx$ and C_1 is a constant of integration.

The gravitational capture of monopoles by the black holes effectively takes place as long as the capture radius is larger than the mean free path of monopoles. This condition is satisfied for $T < (\beta^2 g_*^{-1} b^2 M_m)^{1/3} M_{Pl} \approx (0.1\beta^2)^{1/3} M_{Pl}$. While the electromagnetic monopole-antimonopole capture stops at some finite temperature, the gravitational capture starts at some high temperature and practically never ends. This is the consequence of the fact that the universe can accommodate larger and larger black holes as time passes, so that the gravitational capture radius grows faster than the mean free path of monopoles. Hence we can neglect the monopole annihilation contribution in eq. (12) (the second term in denominator). We note that in general dropping the second term is a conservative assumption in the sense that we are underestimating the destruction of monopoles. Thus we have

$$\frac{n(t)}{s} = \left(\frac{n(t)}{s}\right)_i e^{-0.1 f\beta^2 (t-t_i)/t_{Pl}}\,, \tag{13}$$

where the initial ratio between the monopole and entropy density is given by eq. (1). We see that the gravitational capture of the monopoles by the black holes exponentially reduces this ratio. We note that the most massive black holes contribute the most, while the contribution of much lighter ones can be neglected. Also, if $t_i \ll t$ we can neglect the term t_i in the exponential. This will be true in practically all the cases of interest here.

Now, we can derive the values of f and β needed for a solution of the monopole problem. The first bound on the monopole abundance comes from the requirement that the universe must be radiation dominated at nucleosynthesis, i.e. $T \sim$MeV. This implies that the ratio between the monopole and entropy density can be at most

$$\left(\frac{n}{s}\right)_{T=\mathrm{MeV}} \leq \frac{1\mathrm{MeV}}{M_m} = 10^{-19}\,. \tag{14}$$

We can derive a more stringent bound by requiring that the mass density of the monopoles today should not exceed the critical density: $M_m n_m < \rho_c$. This yields

$$\left(\frac{n}{s}\right)_{\mathrm{today}} \leq \frac{\rho_c}{M_m s_0} = 10^{-24}\,, \tag{15}$$

where we used the current value of the entropy density $s_0 = 10^3\mathrm{cm}^{-3}$. There are a number of other late time constraints [5]. For example, since monopoles are magnetically charged, they can be accelerated by galactic magnetic fields. In this process the galactic magnetic field gets dissipated. In order to avoid a conflict with observations, the upper limit on monopole number density is

$$n_m < 10^{-20}\mathrm{cm}^{-3}\,. \tag{16}$$

Certainly, the strongest constraint comes from the consequences of monopole induced baryon catalysis in neutron stars [7]:

[†]We note that this assumption requires black holes to form at a range of temperatures or to grow due to accretion.

$$n_m < 10^{-25}\mathrm{cm}^{-3}\,. \tag{17}$$

This implies the bound on a current ratio between the monopole and entropy density:

$$\left(\frac{n}{s}\right)_{\mathrm{today}} < 10^{-28}\,. \tag{18}$$

Substituting eq. (1) and eq. (18) into eq. (13) we can derive

$$f\beta^2 t_f/t_{Pl} \geq 100\,, \tag{19}$$

or, equivalently,

$$f\beta^2 \left(m_{pl}/T_f\right)^2 \geq 100 \tag{20}$$

for our fiducial value of $M_m = 10^{16}$ GeV. This is the necessary condition for a solution of the monopole problem. Here, t_f and T_f denote time and temperature respectively when the process of gravitational capture of monopoles by black holes stops. We now discuss whether or not the universe can reasonably produce black holes that satisfy this condition.

Black holes more massive than 10^{15} gm have lifetimes longer than the present age of the universe. While these black holes are dynamically capable of swallowing the monopoles, the energy density that was in the monopoles remains in the form of mass rather than radiation and hence overcloses the universe. Hence we must consider lighter black holes.

Black holes of mass $M_{bh} < 10^{15}$gm evaporate before the present epoch, thereby converting the monopole energy density to radiation which redshifts away. However, observational constraints from earlier epochs have been previously examined by other authors [15] and found to be more severe. These constraints are sensitive to the assumptions in the model, e.g., model of formation of the black holes, equations of state in different epochs of the early universe, whether or not the universe experiences a period (or multiple periods) of inflation, etc. Ref. [15] shows that black holes with mass over 10^9g could not have been produced with appreciable abundance relative to the total energy density without violating observational constraints due to the cosmic microwave background (CMB), nucleosynthesis, geometry of the universe, etc. On the other hand, black holes less massive than 10^9gm are not seriously constrained by observations: they evaporate completely within 0.1sec, i.e. before nucleosynthesis takes place, and are compatible with results of CMB experiments.

Thus, we work with the constraint that only black holes of $M_{bh} \leq 10^9$gm can be responsible for removing monopoles from the universe. Using Eqs.(20) and (9), together with this lower bound on the black hole mass, we find

$$T_f > \frac{3\times 10^{-13} M_{pl}}{g_*^{1/2} f^{1/2}} = 3\times 10^6 \mathrm{GeV} \left(\frac{f}{0.01}\right)^{-1/2} \left(\frac{g_*}{100}\right)^{-1/2}\,. \tag{21}$$

As an example, we will consider $T_f = 10^9$GeV, or, equivalently $t_f \sim 10^{18} t_{Pl}$, for the final temperature. Then the condition (19) becomes

$$f\beta^2 \geq 10^{-16}\,. \tag{22}$$

We plot the allowed range in Fig. 1.

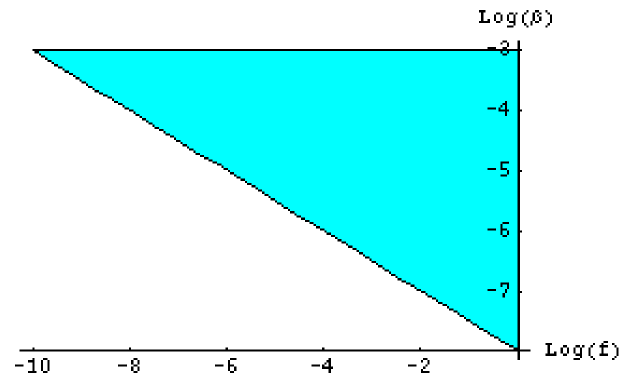

FIG. 1. The allowed parameter range for the solution of the monopole problem (for $M_m = 10^{16}$GeV) lies above the curve $\beta = (10^{-16}/f)^{1/2}$ (shaded region on the Log-Log plot). Parameter f is the fraction of the total energy density of the universe that must be in black holes, while β is the fraction of the horizon mass that represents a typical (average) black hole at that time. Even for values of $f \ll 1$ we can have reasonable values for β.

For example, if $f \sim 10^{-2}$ (1% of the energy density of the universe is in black holes), the allowed range for β is $\beta \geq 10^{-7}$, which means that the mass of a typical black hole at that time should be at least 10^{-7} of the horizon mass. This is not difficult to envision. The horizon mass at $T \sim 10^9$GeV is roughly $M_h \sim 10^{13}$gm, so the average black holes mass should be $M_{bh} \geq 10^6$gm. Note that the number of black holes within a horizon is roughly given by $N_{bh} = f/\beta$, but since β gives only an average description of the system, the real number of black holes could be quite different from N_{bh} defined in this way. We therefore do not use N_{bh} as a reliable quantifier.

We have shown that, as far as the dynamics of the process is concerned, a reasonable number of primordial black holes is capable of capturing monopoles from the early universe. We now discuss problems that might potentially arise.

After absorbing monopoles, a black hole will become charged. In our calculation we assumed that the monopole capture by the black hole is charge blind, i.e. we neglected the electromagnetic interaction between the monopole and black hole. Now, we justify this assumption. Let's consider a black hole which has already captured N_m monopoles and antimonopoles. Most of the

monopoles and antimonopoles inside the black hole will annihilate, leaving $\sqrt{N_m}$ monopoles inside (due to magnetic charge fluctuations). The gravitational mass of the black hole is enhanced to $M_{bh} + N_m M_m$; the magnetic charge of the black hole becomes $\sqrt{N_m} g$, where g is a magnetic charge of the monopole. The gravitational force between this black hole and a monopole outside it is then

$$F_g = G(M_{bh} + N_m M_m) M_m / r^2 \,. \tag{23}$$

The corresponding magnetic force is

$$F_m = \sqrt{N_m} g^2 / r^2 \,. \tag{24}$$

If N_m is large ($N_m > 10^{15}$), gravitational force is always greater than the magnetic. Even if N_m is small, one can easily check that the gravitational attraction between a monopole of mass 10^{16}GeV (with a unit charge g) and a black hole that captured N_m monopoles is larger than the electromagnetic repulsion between them for any black hole with

$$M_{bh} > 10^4 \sqrt{N_m} M_{Pl}, \tag{25}$$

a condition which is easily satisfied for the black holes of interest.

Another problem that might arise in this scenario can come from magnetic charge fluctuations: a black hole that captures N_m monopoles and anti-monopoles will typically have accumulated a residual net magnetic charge of $\sqrt{N_m}$. The question is, what happens to this magnetic charge? One possibility would be that black holes Hawking-radiate monopoles. A monopole is a highly coherent state of many gauge quanta and emission of a monopole by a black hole will be highly suppressed. Even if this process is somehow allowed [17], the Hawking temperature of a black hole becomes of order of a monopole mass only at the end of evaporation, and a black hole could radiate much less monopoles than originally eaten. Generically, since the Hawking radiation can not violate the gauge symmetry, a black hole can not evaporate completely. Instead it leaves a remnant — an extreme magnetically charged Reissner-Nordstrom black hole. The mass of the (non-rotating) remnant M_r must be greater than the magnetic charge Q_m of the black holes, $M_r \geq Q_m$ (in appropriate units), or otherwise the remnant would be a naked singularity. One has to check whether these remnants violate some of the observational constraints. Due to the attractive force between the remnants with opposite charges, their number density will be reduced (charges annihilate and the rest of the mass evaporates freely). However, here we adopt the most conservative scenario in which all the black holes which participate in accreting monopoles leave massive remnants. The mass density $\sqrt{N_m}$ that remains trapped in the extremal Reissner-Nordstrom black holes could potentially be problematic. The energy density in these remnants must be less than that of ordinary radiation at the time of primordial nucleosynthesis, in order for predictions of element abundances to be unaffected. Here we will compare the residual number of monopoles inside black holes and ensure that it is smaller than the number of monopoles allowed by the nucleosynthesis bound in Eq.(14). The total number of monopoles within the horizon at temperature T is $N_{tot} = M_{\rm tot}/M_m$, or, using Eq.(5),

$$N_{tot} \sim 10^{-2} \frac{n_m}{s} \left(\frac{M_{Pl}}{T} \right)^3 \,. \tag{26}$$

We note that this number is proportional to n_m/s. If all of the monopoles within horizon are eaten by black holes at some temperature (e.g. $T \sim 10^9$GeV), then the remnant monopole mass inside the black holes is $\sqrt{N_m} \propto \sqrt{\left(\frac{n_m}{s}\right)_i}$ given in Eq.(1). On the other hand, the allowed number of monopoles is proportional to $N_{\rm allowed} \propto (n_m/s)_{\rm MeV}$ given in Eq.(14). Thus, we can calculate that at $T \sim 10^9$GeV, there are $N_m \sim 10^{16}$ monopoles inside a horizon volume. Practically all of them are eaten by the black holes. Mass density of $\sqrt{N_m} \sim 10^8$ monopoles can not be eliminated (under our conservative assumptions). However, the number of monopoles that the universe can tolerate is $N_m \sim 10^9$. Thus, the mass density of the remnants does not violate observational constraints.

Later, there can be further annihilation of monopole/antimonopole pairs inside galaxies and clusters of galaxies where these objects can clump; hence we shy away from extrapolating the freeze-out density in Eq.(1) (or Eq.(4)) all the way to the current epoch to estimate the mass density of remnants today and then applying the constraint of Eq(15).

In conclusion, we have presented a possible solution to the cosmological monopole problem in which primordial black holes efficiently eliminate the magnetic monopoles from our universe. Under very conservative assumptions, black holes of mass

$$M_{bh} < 10^9 \text{gm} \tag{27}$$

are dynamically capable of solving the cosmological monopole problem, while their required abundance does not violate any of the observational constraints. We did not assume any particular model for primordial black holes formation. Instead, we derived the bounds on abundance of black holes that can reasonably exist in our universe and are capable of removing monopoles. Black holes have a large cross-section for capturing non-relativistic monopoles and the parameter space for solving the monopole problem is not very restrictive. In a similar manner, black holes can in principle remove other unwanted massive relics from the early universe [18].

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